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THOMSON REUTERS

[Patil, 6(4): April 2019] ISSN 2348 – 8034
DOI- 10.5281/zenodo.2654618 Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

MAGNETO CONVECTION FLOW AND HEAT TRANSFER THROUGH A VERTICAL CHANNEL ALONG WITH SOURCE OR SINK AND RADIATION EFFECT

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ABSTRACT

The problem of hydro magnetic fully developed laminar mixed convective flow in a vertical channel with symmetric and asymmetric wall heating conditions in the presence of radiation effect is considered. The velocity and the temperature fields are obtained analytically by perturbation series method which employees a perturbation parameter $\varepsilon = Br\lambda$ proportional to the Brinkman number. The effects of the various parameters such as Brinkman number Br, Hartmann number M, heat source or heat sink ϕ and radiation effect F are illustrated graphically and discussed. It is observed that the effect of M is to decrease the velocity and temperature for positive λ and is to increase for negative λ for both isoflux-isothermal and isothermal-isoflux wall heating conditions.

Keywords: *Magneto Convection, Brinkman number, Hartmann number, Perturbation Series method, Thermal radiation*

I. INTRODUCTION

A combined free and forced convection flow of an electrically conducting and heat-generating or absorbing fluid in a channel in the presence of a transverse magnetic field is of special technical significance because of its frequent occurrence in many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, thermal insulations and petroleum reservoirs. This type of problems also arises in electronic packages, microelectronic devices during their operations.

Laminar mixed convection in a vertical plane channel has been investigated by A. Barletta and M. Celli [1] taking into account the effect of an external uniform magnetic field orthogonal to the flow direction to prove that heat flux due to viscous dissipation and due to joule heating at the isothermal walls are monotonic increasing functions of M. The effect of Magnetic field M in the presence of heat generation and heat absorption increases the rate of heat transfer in the case of Isoflux-Isothermal wall heating conditions and decreases in the case of Isothermal- Isoflux has been studied by J. C Umavathi et al [2]. Finite difference technique was employed by J. C Umavathi et al [3] to study the effect of different parameters governing the flow and heat transfer for both open and short circuits. Patil Mallikarjun B [4] has discussed the problem of steady laminar mixed convection flow in an infinite vertical channel with applied magnetic field in the presence of viscous and ohmic dissipation. M Abd-El Aziz [5] observed that at high temperature differences, applying a uniform magnetic field to the flow can give an acceptable accurate velocity distribution in despite of using Boussinesq approximation which actually gives substantial errors and also discussed that the increase in the radiation parameter, increases wall couple stress and the heat transfer rate and greatly decreases the friction factor of a micro polar fluid. A. J Chamkha [6] studied the reversal flow near the walls of asymmetric channel wall temperatures and mixed wall thermal conditions and proved that the zone of assured reversal flow was found to increase because of the presence of the magnetic field or heat generation effects or both. J. C Umavathi and M. S Malashetty [7] have proved that for asymmetric wall heating conditions the viscous dissipation enhances the effect of flow reversal in the case of downward flow whereas it counters this effect in the case of upward flow. A.Barletta et al [8] also studied mixed convection with heating effects in a vertical porous annulus with a radially varying magnetic field and proved that the velocity and temperature profiles depend in general on the choice of the reference temperature. The study of MHD flow by Ali J Chamka [9] , of a uniformly stretched vertical permeable surface in the presence of heat generation /absorption and a chemical reaction has shown that the fluid velocity increased during a generative chemical reaction and decreased during a destructive one.

Also, he proved that the heat generation effects increased the fluid velocity while the heat absorption effects decreased it.

J.P Garandet et al [10] did an asymptotic analysis to study the buoyancy driven convection in a uniform magnetic field.

II. MATHEMATICAL FORMULATION

Consider steady, laminar, and hydro magnetic fully developed flow in a parallel plate vertical channel. The physical configuration is described in Fig. 1

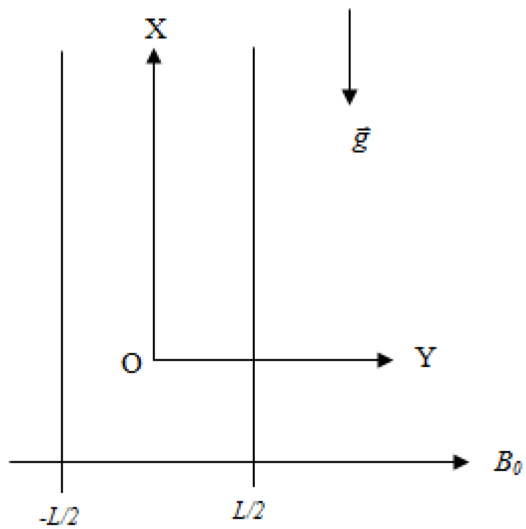

Fig.1 Physical configuration

A constant magnetic field of strength B_0 is applied across the channel. Cartesian co-ordinate system is chosen with the transverse coordinate Y and the coordinate in the direction parallel to the walls is X . The origin of the axes is such that the channel walls are at positions $Y = -L/2$ and $Y = L/2$ .The thermal conductivity, the dynamic viscosity and the thermal expansion coefficient are considered as constant.

The Oberbeck-Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density is assumed to depend upon the temperature according to the equation of state

$$\rho = \rho_0 \left(1 - \beta\left(T - T_0\right)\right) \tag{1}$$

The condition of fully developed flow implies that $\partial U / \partial X = 0$. Then, since the velocity field U is solenoidal, one obtains $\partial V / \partial Y = 0$. As a consequence, the velocity component V is constant in any channel section and is equal to zero at the channel walls, so that V must be vanishing at any position. The Y -momentum balance equation can be expressed as $\partial P / \partial Y = 0$ where $P = p + \rho_0\, g\, X$ is the difference between the pressure and the hydrostatic pressure. Therefore P is depends only on X and the Y momentum balance equation is given by

$$g\beta\left(T - T_0\right) - \frac{1}{\rho_0}\frac{dP}{dX} + \nu\frac{d^2U}{dY^2} - \frac{\sigma_e B_0^2 U}{\rho_0} - \frac{\sigma_e B_o E_o}{\rho_o} = 0 \quad . \tag{2}$$

 ISSN 2348 – 8034
Impact Factor- 5.070

The walls of channel are considered to be isothermal. In particular, the temperature of the boundary at the left wall $Y=-L/2$ is T_1, and the right wall $Y=L/2$ is T_2, with $T_2 \geq T_1$. These wall temperatures are compatible with the equation (2) only when dP/dX is independent of X and is equal to constant A

$$\frac{dP}{dX}=A \tag{3}$$

Differentiating equation (2) with respect to X, and using the equation (3) one obtains

$$\frac{\partial T}{\partial X}=0 \tag{4}$$

which shows that the temperature also depends only on Y.

By taking into account the effects of viscous and ohmic dissipation and the heat source or sink, the energy balance equation can be written as

$$\alpha\frac{d^2T}{dY^2}+\frac{\nu}{C_p}\left(\frac{dU}{dY}\right)^2+\frac{\sigma_e B_0^2}{\rho_0 C_p}U^2+\frac{\sigma_e E_0{}^2}{\rho_0 C_p}+\frac{2\sigma_e E_0 B_0}{\rho_0 C_p}\pm\frac{Q(T-T_0)}{\rho_0 C_p}-\frac{1}{\rho_0 C_P}\frac{dq_R}{dY}=0 \tag{5}$$

Equations (2) and (5) allow one to obtain a differential equation for U, in the form of

$$\frac{d^4U}{dY^4}=\left(\frac{\sigma_e B_0^2}{\mu}\mp\frac{Q}{K}\right)\frac{d^2U}{dY^2}+\left(\frac{2\beta g\sigma_e E_0 B_0}{\nu K}\mp\frac{Q\sigma_e B_0{}^2}{\mu K}\right)U+\frac{\sigma_e B_0^2\beta g}{\alpha C_p \mu}U^2$$
$$+\frac{\beta g}{\alpha C_p}\left(\frac{dU}{dY}\right)^2+\frac{\beta g\sigma_e E_0{}^2}{\nu K}\pm\frac{Q\sigma_e E_0 B_0}{\mu K}\pm\frac{QA}{\mu K}-\frac{g\beta}{K\nu}\frac{dq_R}{dY} \tag{6}$$

The boundary conditions on U are both the no slip conditions

$$U=0 \quad \text{at} \quad Y=\pm\frac{L}{2} \tag{7}$$

and those induced by the thermal boundary conditions on T and by equations (2) and (3) are

$$\frac{d^2U}{dY^2}=\frac{A}{\mu}-\frac{\beta g(T_1-T_0)}{\nu}+\frac{\sigma_e E_0 B_0}{\mu}+\frac{\sigma_e B_0{}^2U}{\mu} \quad \text{at} \quad Y=-\frac{L}{2}$$
$$\frac{d^2U}{dY^2}=\frac{A}{\mu}-\frac{\beta g(T_2-T_0)}{\nu}+\frac{\sigma_e E_0 B_0}{\mu}+\frac{\sigma_e B_0{}^2U}{\mu} \quad \text{at} \quad Y=\frac{L}{2} \tag{8}$$

The following quantities are employed for writing equations (5) to (8) in the dimensionless form

$$u=\frac{U}{U_0};\quad \theta=\frac{T-T_0}{\Delta T};\quad y=\frac{Y}{D};\quad Gr=\frac{g\beta\Delta T D^3}{\nu^2};\ \mathrm{Re}=\frac{U_0 D}{\nu};\ \Pr=\frac{\nu}{\alpha};\ F^2=\frac{CD^2}{K}$$

$$Br=\frac{\mu U_0^2}{K\Delta T};\ \phi=\frac{QD^2}{K};\ \ E=\frac{E_0}{U_0B_0};\ M^2=\frac{\sigma_e B_0^2 D^2}{\mu};\ \ \lambda=\frac{Gr}{\mathrm{Re}};\ \ R_T=\frac{T_2-T_1}{\Delta T};\sigma^2=\frac{D^2}{K} \tag{9}$$

The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0=-\frac{AD^2}{48\mu};\qquad T_0=\frac{T_1+T_2}{2}. \tag{10}$$

The temperature difference ΔT is given either by

$$\Delta T=T_2-T_1\qquad \text{if}\quad T_1\neq T_2\qquad \text{or} \tag{11}$$

$$\Delta \mathrm{T}=\frac{\nu^2}{C_p D^2}\qquad \text{if}\quad T_1=T_2 \tag{12}$$

The dimensionless parameter R_T becomes zero for symmetric heating $(T_1=T_2)$ and one for asymmetric heating $(T_1<T_2)$. Substituting the equation (9) into the equations (5) to (8) yields the following dimensionless equations.

$$\frac{d^2\theta}{dy^2}=-Br\left(\frac{du}{dy}\right)^2-M^2E^2Br-2M^2E\,Br\,u-M^2Br\,u^2\mp(\phi-F^2)\theta \tag{13}$$

$$\frac{d^4u}{dy^4}-\left(M^2\mp\phi+F^2\right)\frac{d^2u}{dy^2}=M^2\lambda\,Br\left(E+u\right)^2+\lambda\,Br\left(\frac{du}{dy}\right)^2\pm M^2(\phi-F^2)u$$
$$\pm M^2(\phi-F^2)E\mp 48(\phi-F^2) \tag{14}$$

Four boundary conditions are

$$u=0\quad \text{at}\quad y=\pm\frac{1}{4} \tag{15}$$

$$\frac{d^2u}{dy^2}=-48+\frac{R_T\lambda}{2}+M^2E\quad \text{at}\quad y=-\frac{1}{4}$$
$$\frac{d^2u}{dy^2}=-48-\frac{R_T\lambda}{2}+M^2E\quad \text{at}\quad y=\frac{1}{4} \tag{16}$$

Using equations (9) and (10) in equation (2) one obtains

$$\theta=-\frac{1}{\lambda}\left(48-M^2u+\frac{d^2u}{dy^2}-M^2E\right) \tag{17}$$

The dimensionless temperature θ can be evaluated either by integrating equation (13) or by using equation (17).

 ISSN 2348 – 8034
Impact Factor- 5.070

If the viscous dissipation is negligible so that $Br = 0$, the dimensionless temperature θ and dimensionless velocity u are uncoupled. In this case solutions of equation (14) using boundary conditions (15) and (16) is

$$u = \left(\frac{48}{M^2} - E\right)\left(1 - \frac{CoshMy}{CoshM/4}\right) + \frac{\lambda R_T}{2(M^2 + (\phi - F^2))}\left(\frac{Sin\sqrt{\phi - F^2}\,y}{Sin\sqrt{\frac{\phi - F^2}{4}}} - \frac{SinhMy}{Sinh\frac{M}{4}}\right)$$

for the case of heat source and (18)

$$u = \left(\frac{48}{M^2} - E\right)\left(1 - \frac{CoshMy}{CoshM/4}\right) + \frac{\lambda R_T}{2(M^2 - (\phi + F^2))}\left(\frac{Sinh\sqrt{\phi + F^2}\,y}{Sinh\sqrt{\frac{\phi + F^2}{4}}} - \frac{SinhMy}{Sinh\frac{M}{4}}\right) \quad (19)$$

for the case of heat sink.

By substituting these velocity fields in equation (17), we get the temperature field for both the cases of heat source and heat sink as,

$$\theta = \frac{R_T}{2}\frac{Sin\sqrt{\phi - F^2}\,y}{Sin\sqrt{\frac{\phi - F^2}{4}}} \quad (20)$$

$$\theta = \frac{R_T}{2}\frac{Sinh(\sqrt{\phi + F^2})y}{Sinh\sqrt{\frac{\phi + F^2}{4}}} \quad (21)$$

If the electrical conductivity is negligible i.e. $E = 0$, then the velocity field for both the cases reduces to

$$u = \frac{48}{M^2}\left(1 - \frac{CoshMy}{Cosh\frac{M}{4}}\right) + \frac{\lambda R_T}{2(M^2 + \phi - F^2)}\left(\frac{Sin\sqrt{\phi - F^2}\,y}{Sin\sqrt{\frac{\phi - F^2}{4}}} - \frac{SinhMy}{Sinh\frac{M}{4}}\right) \quad (22)$$

$$u = \frac{48}{M^2}\left(1 - \frac{CoshMy}{Cosh\frac{M}{4}}\right) + \frac{\lambda R_T}{2(M^2 - (\phi + F^2))}\left(\frac{Sinh\sqrt{\phi + F^2}\,y}{Sinh\sqrt{\frac{\phi + F^2}{4}}} - \frac{SinhMy}{Sinh\frac{M}{4}}\right) \quad (23)$$

The temperature equation remains same as given by the above equations (20) and (21). When the parameters $\phi = E = Br = F = 0$, velocity and temperature fields reduces to

$$u = \frac{48}{M^2}\left(1 - \frac{CoshMy}{CoshM/4}\right) + \frac{2\lambda\, R_T}{M^2}\left(y - \frac{SinhMy}{4SinhM/4}\right) \tag{24}$$

$$\theta = 2R_T y \tag{25}$$

In the absence of applied magnetic field, electrical conductivity and internal heat source or sink coefficient the expression of velocity is,

$$u = \left(\frac{R_T\lambda}{3}y + 24\right)\left(\frac{1}{16} - y^2\right) \tag{26}$$

and the temperature equation is same as given in equation (25).

In the case of asymmetric heating, when buoyancy forces are dominating i.e., when $\lambda \to \pm\infty$, equations (18) and (19) gives,

$$\frac{u}{\lambda} = \frac{1}{2(M^2 + \phi - F^2)}\left(\frac{Sin\sqrt{\phi - F^2}\,y}{Sin\sqrt{\phi - F^2}/4} - \frac{SinhMy}{SinhM/4}\right) \tag{27}$$

$$\frac{u}{\lambda} = \frac{1}{2(M^2 - (\phi + F^2))}\left(\frac{Sinh\sqrt{\phi + F^2}\,y}{Sinh\sqrt{\phi + F^2}/4} - \frac{SinhMy}{SinhM/4}\right) \tag{28}$$

Similarly equations (24) and (26) becomes,

$$\frac{u}{\lambda} = \frac{2}{M^2}\left(y - \frac{SinhMy}{4SinhM/4}\right) \tag{29}$$

$$\frac{u}{\lambda} = \frac{y}{3}\left(\frac{1}{16} - y^2\right) \tag{30}$$

which is Batchelor's velocity profile for free convection.

Solutions of equations (14) and (17) for viscous fluid in the absence of applied magnetic field, electrical conductivity, heat source or sink coefficient and buoyancy force leads to the Hagen-Poiseuille velocity profile

$$u = 24\left(\frac{1}{16} - y^2\right) \tag{31}$$

$$\theta = -192\ Br\ y^4 + 2R_T y + \frac{3Br}{4} \tag{32}$$

If buoyancy forces are not considered, then $\lambda = 0$ and viscous term is dominating i.e. $Br \neq 0$ a purely forced convection occurs. In this case the solutions of velocity and temperature field becomes,

$$u = \left(\frac{48}{M^2} - E\right)\left(1 - \frac{CoshMy}{CoshM/4}\right) \tag{33}$$

for both the cases of heat source or heat sink.

$$\theta = C_1 Cos\sqrt{\phi - F^2}\,y + C_2 Sin\sqrt{\phi - F^2}\,y + l_1 Cosh2My + l_2 CoshMy + l_3 \tag{34}$$

where, $l_1 = -Br\left(M^2E^2 - 96E + \frac{2304}{M^2}\right)\frac{Cosh2My}{(4M^2 + \phi - F^2)}$

$l_2 = -Br\left(96E - \frac{4608}{M^2}\right)\frac{CoshMy}{(M^2 + \phi - F^2)}$; $\quad l_3 = -\frac{2304Br}{M^2(\phi - F^2)}$

$C_1 = -\frac{1}{Cos\left(\frac{\sqrt{\phi - F^2}}{4}\right)}\left(l_1 Cosh\frac{M}{2} + l_2 Cosh\frac{M}{4} + l_3\right)$; $\quad C_2 = \frac{R_T}{2Sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)}$

for the case of heat source and

$$\theta = C_1 Cosh\sqrt{\phi + F^2}\,y + C_2 Sinh\sqrt{\phi + F^2}\,y + l_1 Cosh2My + l_2 CoshMy + l_3 \tag{35}$$

where, $l_1 = -Br\left(M^2E^2 - 96E + \frac{2304}{M^2}\right)\frac{1}{(4M^2 - (\phi + F^2))Cosh^2 P_3}$

$l_2 = -Br\left(96E - \frac{4608}{M^2}\right)\frac{1}{D_9 CoshP_3}$; $\quad l_3 = \frac{2304Br}{M^2(\phi - F^2)}$

$C_1 = -\frac{1}{CoshP_4}(l_1 CoshP_1 + l_2 CoshP_3 + l_3)$; $\quad C_2 = \frac{R_T}{2SinhP_4}$

for the case of heat sink.

III. SOLUTIONS

Equation (14) is nonlinear because of viscous and Ohmic dissipations and it is difficult to find the closed form solution. Thus perturbation series method is employed by defining the dimensionless parameter

$$\varepsilon = Br\,\lambda = Re\,Pr\,\frac{\beta g D}{C_p} \tag{36}$$

as the perturbation parameter. Then the temperature field is obtained using equation (17). The solution of velocity field can be expressed by the perturbation expansion

$$u(y) = u_0(y) + \varepsilon\,u_1(y) + \varepsilon^2 u_2(y) + \cdots\cdots\cdots = \sum_{n=0}^{\infty}\varepsilon^n u_n(y) \tag{37}$$

The second and higher order terms of ε give a correction to $u_0,\ \theta_0$ accounting for the viscous and Ohmic dissipation effects. Substituting equation (2) in equation (14) to (16) and equating the coefficients of like powers of ε on both sides, one obtains the boundary value problem for $n = 0$ as

Isothermal-Isothermal $(T_1 - T_2)$ walls

$$\frac{d^4u_0}{dy^4}=\left(M^2\mp\phi-F^2\right)\frac{d^2u_0}{dy^2}\pm M^2(\phi-F^2)u_0\pm M^2(\phi-F^2)E\mp 48(\phi-F^2) \quad (38)$$

for the cases of heat source or heat sink.

$$u_0=0 \quad \text{at} \quad y=\pm\frac{1}{4} \quad (39)$$

$$\frac{d^2u_0}{dy^2}=-48+\frac{R_T\lambda}{2}+M^2E \quad \text{at} \quad y=-\frac{1}{4}$$
$$\frac{d^2u_0}{dy^2}=-48-\frac{R_T\lambda}{2}+M^2E \quad \text{at} \quad y=\frac{1}{4} \quad (40)$$

Equation (3) is ordinary linear differential equation and its exact solutions can be found. The solutions of equation (3) are same as we obtain in the case of $Br=0$. The solution of equation (3) using equations (4) and (5) are

$$u_0=C_1CoshMy+C_2SinhMy+C_3Cos\sqrt{\phi-F^2}\,y+C_4Sin\sqrt{\phi-F^2}\,y-E+\frac{48}{M^2} \quad (41)$$

for the case of heat source and

$$u_0=C_1CoshMy+C_2SinhMy+C_3Cosh\sqrt{\phi-F^2}\,y+C_4Sinh\sqrt{\phi-F^2}\,y-E+\frac{48}{M^2} \quad (42)$$

for the case of heat sink. The differential equation for $n=1$ and its boundary conditions become

$$\frac{d^4u_1}{dy^4}=\left(M^2\mp\phi-F^2\right)\frac{d^2u_1}{dy^2}\pm M^2(\phi-F^2)u_1+\left(\frac{du_0}{dy}\right)^2+M^2\left(E+u_0\right)^2 \quad (43)$$

for the cases of heat source and sink.

$$u_1=0 \quad \text{at} \quad y=\pm\frac{1}{4} \quad (44)$$

$$\frac{d^2u_1}{dy^2}=0 \quad \text{at} \quad y=\pm\frac{1}{4} \quad (45)$$

The solutions of equation (8) and (9) by applying the conditions given in equations (9) and (10) are,

$$\begin{aligned}u_1=&\,C_5CoshMy+C_6SinhMy+C_7Cos\sqrt{\phi-F^2}\,y+C_8Sin\sqrt{\phi-F^2}\,y+l_1Cosh2My\\&+l_2Cos2\sqrt{\phi-F^2}\,y+l_3Sinh2My+l_4Sin2\sqrt{\phi-F^2}\,y-l_5SinhMyCos\sqrt{\phi-F^2}\,y\\&+l_6CoshMySin\sqrt{\phi-F^2}\,y-l_7CoshMyCos\sqrt{\phi-F^2}\,y+l_8SinhMySin\sqrt{\phi-F^2}\,y\\&+l_9ySinhMy+l_{10}yCoshMy-l_{11}ySin\sqrt{\phi-F^2}\,y+l_{12}yCos\sqrt{\phi-F^2}\,y-l_{13}\end{aligned} \quad (46)$$

for the case of heat source and

$$u_1 = C_5 CoshMy + C_6 SinhMy + C_7 Cosh\sqrt{\phi+F^2}\,y + C_8 Sinh\sqrt{\phi+F^2}\,y + l_1 Cosh2My + l_2 Sinh2My + l_3 Cosh2\sqrt{\phi+F^2}\,y + l_4 Sinh2\sqrt{\phi+F^2}\,y + l_5 Sinh(M-\sqrt{\phi+F^2})y + l_6 Cosh(M-\sqrt{\phi+F^2})y + l_7 Sinh(M+\sqrt{\phi+F^2})y + l_8 Cosh(M+\sqrt{\phi+F^2})y + l_9 yCoshMy + l_{10} ySinhMy + l_{11} yCosh\sqrt{\phi+F^2}\,y + l_{12} ySinh\sqrt{\phi+F^2}\,y + l_{13} \quad (47)$$

for the case of heat sink.

Evaluation of exact solutions for $n=2$ becomes complicated and hence neglecting the terms of $n=2$ and onwards, the solution of equation (14) is

$$u = u_0 + \varepsilon u_1. \quad (48)$$

The dimensionless temperature field is obtained from the equation (17) by substituting the solutions of u_0 and u_1 in equation (13) for both the cases of heat source or sink and is given by

$$\theta = \frac{1}{\lambda}\begin{pmatrix} \varepsilon(M^2+\phi-F^2)(C_7+l_{12}y)Cos\sqrt{\phi-F^2}\,y + (M^2+\phi-F^2)(C_4+\varepsilon C_8-\varepsilon l_{11}y)Sin\sqrt{\phi-F^2}\,y - \\ 3\varepsilon M^2(l_1 Cosh2My + l_3 Sinh2My) + \varepsilon(M^2+4(\phi-F^2))(l_2 Cos2\sqrt{\phi-F^2}\,y + l_4 Sin2\sqrt{\phi-F^2}\,y) \\ -\varepsilon(l_5(\phi-F^2)+2l_6 M\sqrt{\phi-F^2})(SinhMyCos\sqrt{\phi-F^2}\,y + \varepsilon(l_6(\phi-F^2)-2l_5 M\sqrt{\phi-F^2}) \\ CoshMySin\sqrt{\phi-F^2}\,y - \varepsilon(l_7(\phi-F^2)+2l_8 M\sqrt{\phi-F^2})CoshMyCos\sqrt{\phi-F^2}\,y + \varepsilon(l_8(\phi-F^2) \\ -2l_7 M\sqrt{\phi-F^2})SinhMySin\sqrt{\phi-F^2}\,y - 2\varepsilon M(l_9 SinhMy + l_{10} CoshMy) - 2\varepsilon\sqrt{\phi-F^2} \\ (l_{11}Cos\sqrt{\phi-F^2}\,y + l_{12}Sin\sqrt{\phi-F^2}\,y) - \varepsilon l_{13}M^2 \end{pmatrix} \quad (49)$$

$$\theta = \frac{1}{\lambda}\begin{pmatrix} (M^2-(\phi+F^2))(C_3+\varepsilon C_7+\varepsilon l_{11}y)Cosh\sqrt{\phi+F^2}\,y + (M^2-(\phi-F^2))(C_4+\varepsilon C_8+\varepsilon l_{12}y) \\ Sinh\sqrt{\phi+F^2}\,y - 3\varepsilon M^2(l_1 Cosh2My + l_2 Sinh2My) - \varepsilon(M^2-4(\phi+F^2))(l_3 Cosh2\sqrt{\phi+F^2}\,y \\ +l_4 Sinh2\sqrt{\phi+F^2}\,y) + \varepsilon((\phi+F^2)-2M\sqrt{\phi+F^2})(l_5 Sinh(M-\sqrt{\phi-F^2})y + l_6 Cosh(M - \\ \sqrt{\phi+F^2})y + \varepsilon((\phi+F^2)+2M\sqrt{\phi+F^2})(l_7 Sinh(M+\sqrt{\phi+F^2})y + l_8 Cosh(M+\sqrt{\phi+F^2}\,y) \\ +\varepsilon 2M(l_9 SinhMy + l_{10} CoshMy) + 2\sqrt{\phi+F^2}(l_{11}Sinh\sqrt{\phi+F^2}\,y + l_{12}Cosh\sqrt{\phi+F^2}\,y) - \varepsilon l_{13}M^2 \end{pmatrix} \quad (50)$$

Isoflux-isothermal $(q_1 - T_2)$ walls

The non-dimensional quantities of thermal boundary conditions at the channel walls are,

$$q_1 = -K\frac{dT}{dY} \quad \text{at} \quad Y = -\frac{L}{2}$$
$$T = T_2 \quad \text{at} \quad Y = \frac{L}{2} \tag{51}$$

The dimensionless form of equation (16) can be obtained by using the equation (9) with $\Delta T = q_1 D / K$ to give

$$\frac{d\theta}{dy} = -1 \quad \text{at} \quad y = -\frac{1}{4}$$
$$\theta = R_{qt} \quad \text{at} \quad y = \frac{1}{4} \tag{52}$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter for the isoflux-isothermal case. Other than the no-slip conditions at the channel walls, two more boundary conditions in terms of U are needed to solve the equation (6). These are the conditions given by equation (17).

Differentiating equation (2) with respect to Y with $dP/dX = A$ gives

$$\frac{d^3U}{dY^3} - \frac{\sigma_e B_0^{\,2}}{\mu}\frac{dU}{dY} + \frac{\beta g}{\nu}\frac{dT}{dY} = 0 \tag{53}$$

Dimensionless form of above equation can be evaluated by using the equation (9),

$$\frac{d^3u}{dy^3} - M^2\frac{du}{dy} + \lambda\frac{d\theta}{dy} = 0 \tag{54}$$

Evaluating the equation (19) at the left wall $(y = -1/4)$ yields

$$\frac{d^3u}{dy^3} - M^2\frac{du}{dy} = \lambda \quad \text{at} \quad y = -\frac{1}{4} \tag{55}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{qt} such that

$$\frac{d^2u}{dy^2} = -48 + \frac{R_{qt}\lambda}{2} + M^2E \quad \text{at} \quad y = -\frac{1}{4} \tag{56}$$

The solutions of velocity field and temperature field can be solved from equations (3), (8), (9) and (19) by using the boundary conditions (4), (5), (10) and (11).

Isothermal-isoflux $(T_1 - q_2)$ walls

The non-dimensional quantities of thermal boundary conditions at the channel walls are,

$$q_2 = -K\frac{dT}{dY} \quad \text{at} \quad Y = \frac{Y}{2}$$

$$T = T_1 \quad \text{at} \quad Y = -\frac{Y}{2} \tag{57}$$

The dimensionless form of above equation can be obtained by using the equation (9) with $\Delta T = q_2 D / K$ to give

$$\frac{d\theta}{dy} = -1 \quad \text{at} \quad y = \frac{1}{4}$$

$$\theta = R_{tq} \quad \text{at} \quad y = -\frac{1}{4} \tag{58}$$

where $R_{tq} = (T_1 - T_0)/\Delta T$ is the thermal ratio parameter for the isothermal-isoflux case. In this case also, the dimensionless form boundary conditions are solved similar to the previous section of isoflux-isothermal walls.

$$\frac{d^3u}{dy^3} - M^2\frac{du}{dy} = \lambda \quad \text{at} \quad y = \frac{1}{4} \tag{59}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{tq} such that

$$\frac{d^2u}{dy^2}\left(\frac{1}{4}\right) = \frac{\lambda\, R_{tq}}{2} - 48 + M^2 E \tag{60}$$

The solutions of velocity field and temperature field can be obtained from these conditions substituting in equations (3), (8), (9) and (16) for the cases of heat source and heat sink up to $O(\varepsilon^1)$.

IV. RESULTS AND DISCUSSION

The problem of laminar magneto convection flow in a vertical channel in the presence of heat source or heat sink and in the presence of thermal radiation is presented graphically and the results are discussed in this section.

Figures 2 to 5 display the effect of velocity and temperature fields for different values of λ and ε. When λ is positive, ε is also positive and the flow is upward and the one other hand the flow is downward when λ and ε are negative. The effect of λ and ε on temperature is not sensible. It is also observed that the effect of λ and ε on u and θ remains the same for open and short circuits.

Figures 6,7,8 and 9 presents velocity and temperature for various values of Hartmann number M for $\lambda = \pm 500$. It shows that the effect of magnetic field on the fluid flow in the channel is to suppress the flow. For λ is positive results in reverse flow near the left wall and negative values of λ produces flow reversal at the right wall. Application of a transverse magnetic field to an electrically conducting fluid gives rise to the Lorentz force, which acts in the direction opposite to that of the fluid causing it to slow down. This drag-like force increases as the strength of the magnetic field (represented by Hartmann number M) increases producing further reductions in the fluid velocity. For the buoyancy aiding flow case (where λ is positive) increasing the Hartmann number reduces the fluid adjacent to the cold left wall causing a flow reversal condition there. This reversed flow phenomenon increases as the strength of the magnetic field increases. For the buoyancy opposing flow case (where λ is negative) the same phenomenon of reversed flow occurs but close to the hot right wall.

Figure 10 illustrates the influence of heat source coefficient ϕ on the velocity profile. It is observed that as ϕ increases velocity and the magnitude is large for $E=-1$ compare to $E=1$ Plots of u and θ are shown in Figures 11 and 12 for different values of Radiation parameter F. Here also as the radiation parameter F increases, the flow is enhanced at the right hot wall whereas flow reversal is observed at the left cool wall. The result is a similar in the case of temperature profile as well. It is seen that the radiation parameter F increases velocity linearly for small values of ε whereas flow reversal is observed for large values of ε .

V. CONCLUSION

The problem of magneto convection flow in an infinite vertical channel with heat source, sink and thermal radiation in the presence of viscous dissipation is discussed. Three different combinations of thermal left-right wall conditions are presented. Analytical solution for the flow and temperature fields with reference to three different special cases are obtained. graphical representations of all the results are presented for different parameters governing the flow and heat transfer. It is observed that the magneto convection parameter increases the velocity and temperature fields and also the additional radiation parameter positively effects in increasing the velocity and temperature fields.

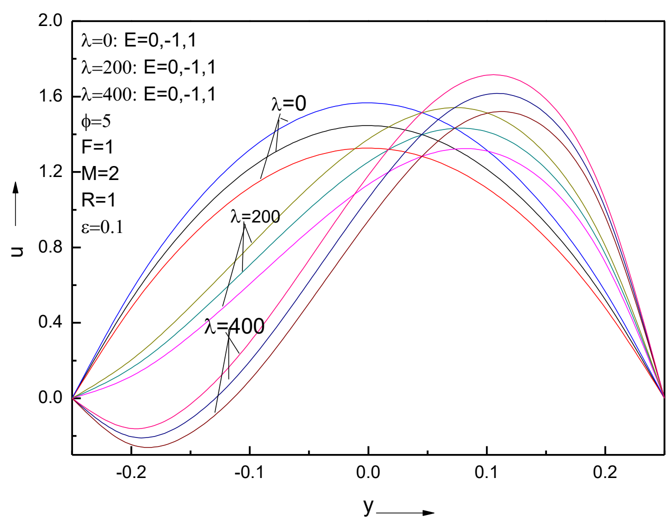

Fig.2: Plots of u versus y in the case of asymmetric heating for different values of λ

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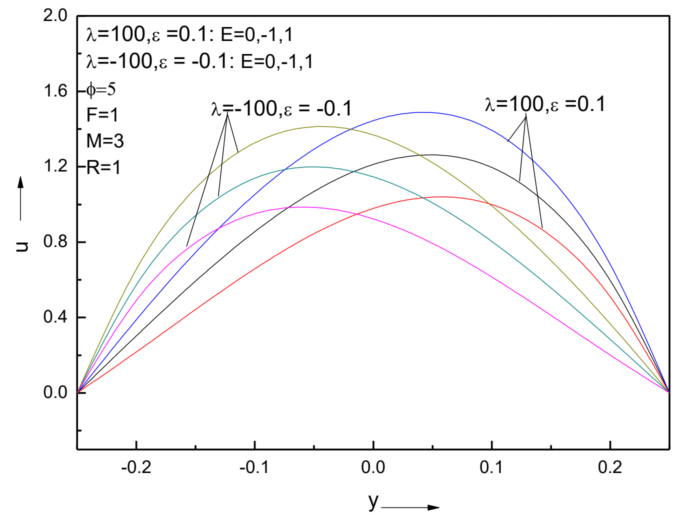

Fig.3: Plots of u versus y in the case of asymmetric heating for different values of λ and ε

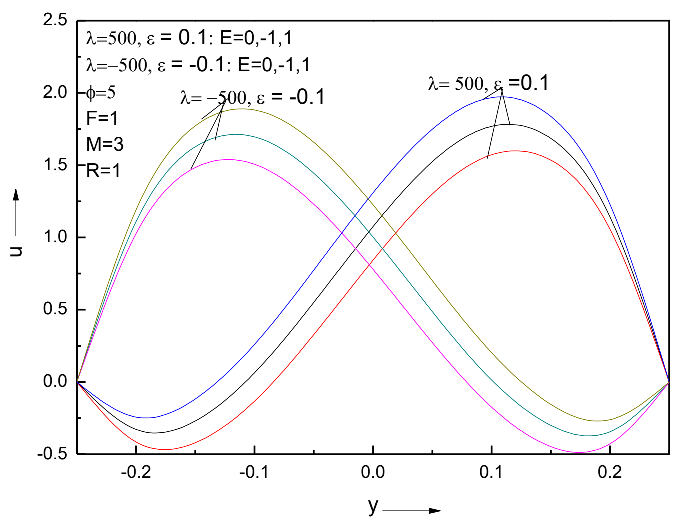

Fig.4: Plots of u versus y in the case of asymmetric heating for different values of λ and ε

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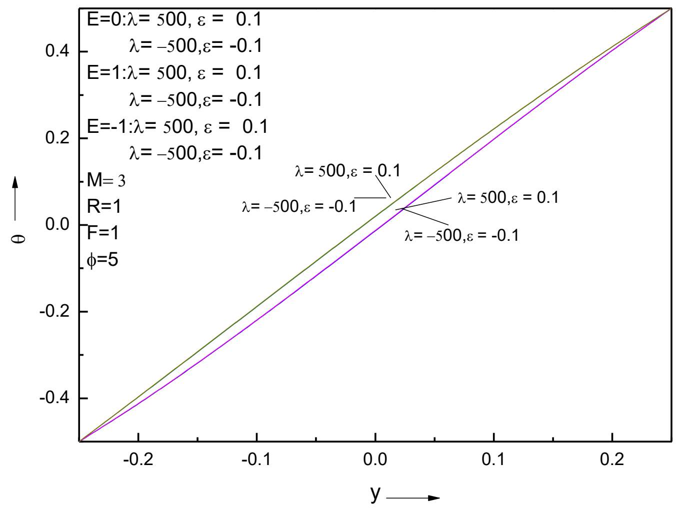

Fig.5: Plots of θ versus y in the case of asymmetric heating for different values of λ and ε

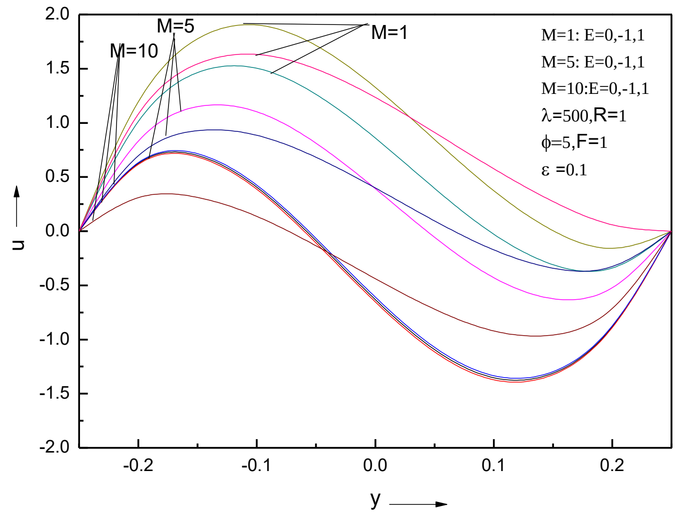

Fig.6: Plots of u versus y in the case of asymmetric heating for different values of Hartman number M

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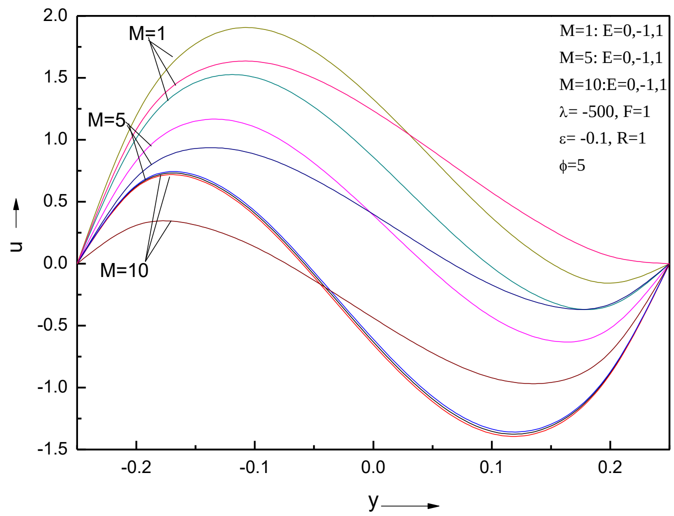

Fig.7: Plots of u versus y in the case of asymmetric heating for different values of Hartman number M

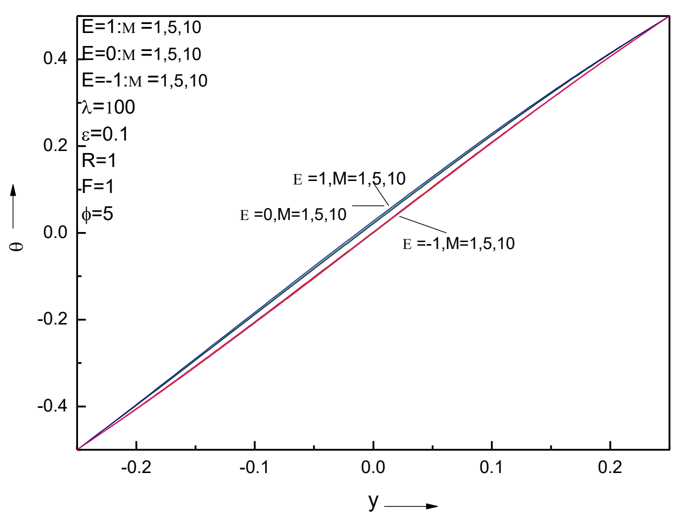

Fig.8: Plots of θ versus y in the case of asymmetric heating for different values of Hartmann number M

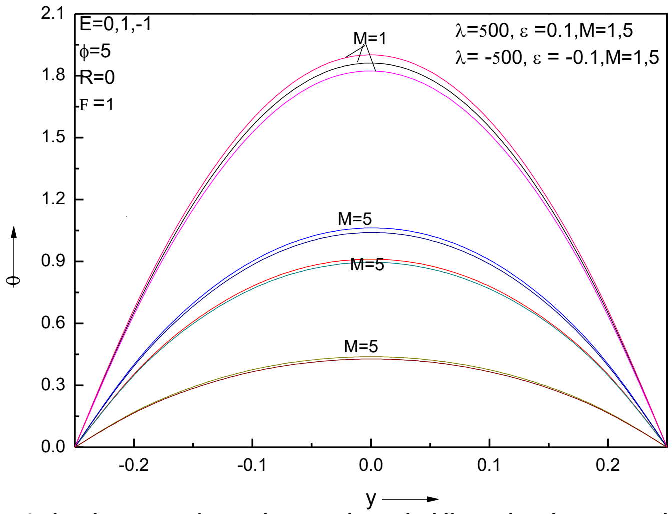

Fig.9: Plots of u versus y in the case of symmetric heating for different values of Hartmann number M

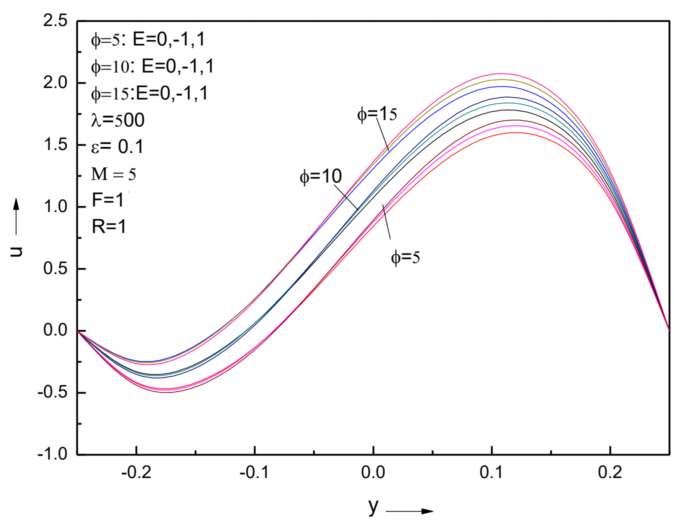

Fig.10: Plots of u versus y in the case of asymmetric heating for different values of heat generation coefficient ϕ

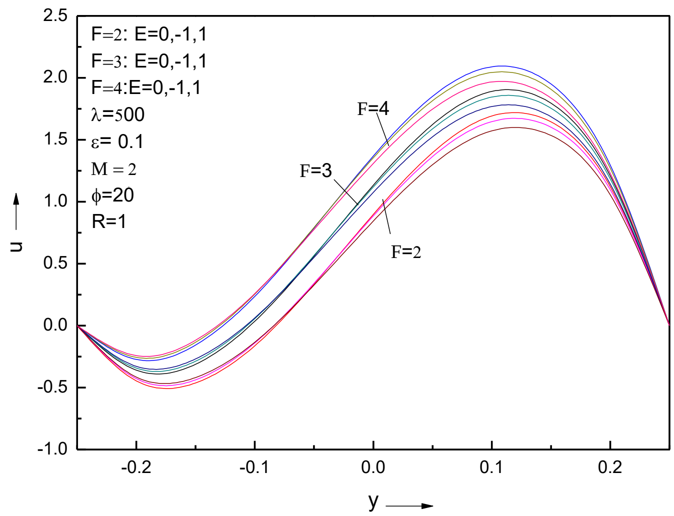

Fig.11: Plots of u versus y in the case of asymmetric heating for different values of radiation parameter F

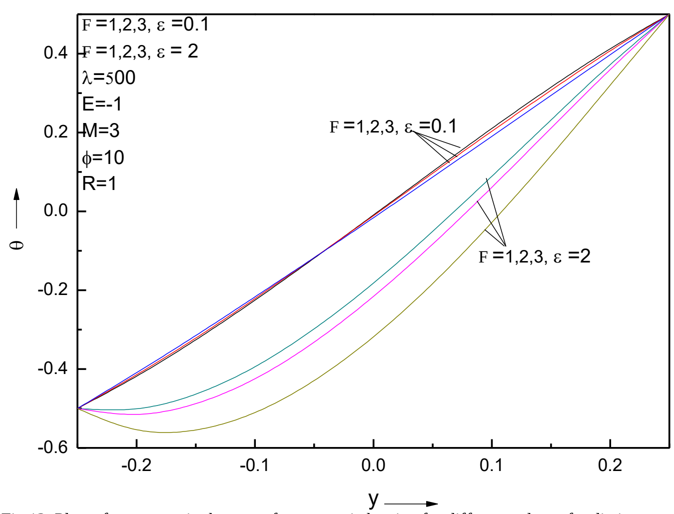

Fig.12: Plots of θ versus y in the case of asymmetric heating for different values of radiation parameter F

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